Low-Complexity Viterbi Metrics applied to Bit-Interleaved COFDM

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Abstract

In this paper, we address the problem of deriving bit-wise metrics for the Viterbi decoding of a bit-interleaved coded modulation. We put an emphasis on Gray-mapped high order QAM constellations, and illustrate the performance in the context of the IEEE 802.11a/g wireless OFDM system. Several bit metric formulas are compared, enabling a performance versus complexity trade-off. It is shown that near-optimum performance can be obtained with reasonable complexity, while a further complexity reduction can be achieved by recursively computing bit metrics for a given QAM symbol, at the expense of about 1dB PER performance loss.

I. Introduction

Bit-Interleaved Coded Modulation (BICM) [3] associated with OFDM [2] is an efficient scheme for transmitting high order constellations on channels with strong multipath. It has been adopted in standards such as IEEE802.11a/g [6] and ETSI HIPERLAN/2. In these standards, the convolutionally coded and punctured bits associated to an OFDM symbol are interleaved prior to Gray mapping on a QAM constellation. Due to the well-known property of OFDM, the multipath channel is equivalent to a set of parallel flat fading channels as long as the multipath spread is shorter than the cyclic prefix duration. At the receiver side, the complex channel gains are estimated and a soft metric is computed for each bit. These bit metrics are depunctured, deinterleaved and fed to the Viterbi decoder.

The computation of bit metrics from the received QAM symbols with the knowledge of the complex channel gains and noise variance is a critical task impacting receiver complexity and performance. For BPSK and QPSK, which can be viewed as trivial QAM, the Maximum Likelihood (ML) decoding of Coded OFDM (COFDM) is presented in [2] and consists of an equalization by multiplying the received symbol by the complex conjugate of the channel estimate, and taking either the real or imaginary part, depending upon the bit. In [3], the Maximum Likelihood bit metrics are derived for any set of symbols, assuming ideal interleaving. A simplification is also proposed, using the log-sum approximation. This approximation amounts to computing for each bit of the symbol and for each binary value, the minimum euclidean distance from the received symbol to the subset of candidate received symbols determined by this bit. Such an approximation is also used in [8]. Both the ideal ML and the log-sum approximation have a complexity proportional to the constellation size. Therefore, when high bit rates and high-order constellations are simultaneously targetted, as in IST project Broadway [7], it becomes useful to seek further simplification of the above bit metrics formulas.

Lower complexity bit metrics were already investigated in the context of Gray-mapped M-ary QAM. In [1], the authors show that only $\sqrt{M}/2$ distance calculations on real numbers are required per bit metric. In [4] and [5], recursive expressions for the Log Likelihood Ratios of a 16-QAM are given. Such an approach makes the complexity proportional to the logarithm of the constellation size. However, in the litterature there lacks a general overview of the performance/complexity trade-off for the various metrics, and an explanation on how to implement them in widespread OFDM systems with multipath.

In this paper, we apply the above-mentioned approximations to Gray-Mapped M-ary QAM BICM-OFDM and detail the equalization and metrics weighting procedure required when operating on a multipath fading channel. Section II presents the derivation of the various metric formulas and section III compares their performance before drawing conclusions in the last section.

II. Optimum and approximated bit metrics

In the following, we reuse the same notations as [9]. For simplicity, the transmit path is illustrated on figure 1 in the specific context of a rate 1/2 convolutionally coded 16-QAM transmission, but the notations are general.

The information bits b_n are coded and interleaved prior to M-ary QAM mapping. The ith bit of the kth transmitted M-QAM symbol s_k is denoted by d_k^i. The channel consists of a complex gain h_k and the addition of a complex white gaussian noise $n_k \sim \mathcal{N}(0, \frac{\sigma^2}{2}) + j\mathcal{N}(0, \frac{\sigma^2}{2})$. The bit metrics are computed from the received symbols y_k.

As stated in [9], since there is a one-to-one correspondence between information words and interleaved codewords $\mathbf{d} \in \mathcal{D}$, where $\mathcal{D}$ is the set of interleaved codewords, the Viterbi decoder achieves Maximum Log-Likelihood codeword estimation under ideal interleaving assumption, by maximizing the following sum:

$$\begin{aligned} \hat{\mathbf{d}} &= \underset{\mathbf{d} \in \mathcal{D}}{\operatorname{argmax}} \log p(\mathbf{y}|\mathbf{d}) & (1) \\ &= \underset{\mathbf{d} \in \mathcal{D}}{\operatorname{argmax}} \log \prod_k p(y_k|s_k) & (2) \\ &\approx \underset{\mathbf{d} \in \mathcal{D}}{\operatorname{argmax}} \sum_k \sum_{i=1}^{B} \log p(d_k^i|y_k) & (3) \end{aligned}$$

where B is defined by $B \hat{=} \log_2(M)$. We denote by $m(k,i)$ the additive Viterbi metric for the ith bit of the kth symbol. Note that for simplicity, we omit in the paper to precise that there are two Viterbi metrics per bit, and we therefore assume that $m(k,i)$ represents either of the two quantities $\log p(d_k^i = b|y_k)$ with $b \in \{0,1\}$.

A first expression $m_1(k,i)$ can be computed (up to an additive constant) using Bayes formula as:

$$m_1(k,i) = \log\left(\sum_{\substack{d_k^j \in \{0,1\} \\ j \neq i}} \exp\left(-\frac{1}{\sigma^2}\left|y_k - h_k s_k(d_k^1, \ldots, d_k^B)\right|^2\right)\right) \quad (4)$$

Equation (4) can be approximated by keeping only the dominant term in the sum, leading to a second metric:

$$m_2(k,i) = -\frac{1}{\sigma^2} \min_{d_k^j \in \{0,1\}, j \neq i} \left|y_k - h_k s_k(d_k^1, \ldots, d_k^B)\right|^2 \quad (5)$$

The ratio $-\frac{1}{\sigma^2}$ can be removed since the noise variance remains constant over the whole codeword. However, we prefer to keep it since it can be used to weight metrics when colored interference is present, which happens for instance in OFDM WLAN systems due to frequency reuse. The straightforward implementation of equations (4) and (5) would require M complex euclidean distance computations per bit metric (since each metric must be computed for two binary values).

Introducing $u_k^i \hat{=} 2d_k^i - 1$ to simplify the notations, it is not required to compute equation (5) for BPSK and QPSK. Actually, it is shown in [2] that ML bit metrics can easily be derived from symbol euclidean metrics using the fact that they are constant modulus constellations. For instance in QPSK with the mapping $s_k = u_k^1 + ju_k^2$, bit metrics are:

$$\begin{aligned} m_2(k,1) &= \frac{1}{\sigma^2} \Re(y_k h_k^*) u_k^1 \text{ if } M = 4 & (6) \\ m_2(k,2) &= \frac{1}{\sigma^2} \Im(y_k h_k^*) u_k^2 \text{ if } M = 4 & (7) \end{aligned}$$

Additional simplification can be obtained without any approximation by replacing y_k by the equalized symbols $z_k = \frac{y_k}{h_k}$ in equations (3) and (5). In this case, (5) can be rewritten as:

$$\log\left(p(d_k^i|z_k)\right) \approx -\frac{|h_k|^2}{\sigma^2} \min_{d_k^j \in \{0,1\}, j \neq i} \left|z_k - s_k(d_k^1, \ldots, d_k^B)\right|^2 \quad (8)$$

We now assume a normalized Gray-mapped M-QAM constellation, in which the mapping of bits d_i onto the symbol s is performed as follows:

$$\Re(s) = \sqrt{\frac{3}{2(M^2-1)}} \sum_{i=1}^{\frac{B}{2}} (-1)^{i-1} 2^{\frac{B}{2}-i+1} \prod_{j=1}^{i} u_j \quad (9)$$

$$\Im(s) = \sqrt{\frac{3}{2(M^2-1)}} \sum_{i=1}^{\frac{B}{2}} (-1)^{i-1} 2^{\frac{B}{2}-i+1} \prod_{j=1}^{i} u_{j+\frac{B}{2}} \quad (10)$$

As mentioned in [1], a Gray mapping associates each bit either to the in-phase or to the quadrature component. Therefore, with the mapping defined by equations (9) and (10), the log-sum approximation can be further simplified without any approximation. For the first $B/2$ bits we define:

$$\underset{1 \leq i \leq B/2}{m_3(k,i)} = -\frac{|h_k|^2}{\sigma^2} \min_{\substack{d_k^j \in \{0,1\}, 1 \leq j \leq \frac{B}{2}, j \neq i \\ d_k^j = 0 \text{ if } \frac{B}{2}+1 \leq j \leq B}} |\Re(z_k) - \Re(s_k)|^2 \quad (11)$$

In equation (11), bits d_k^j, $1 \leq j \leq B/2$ are set to zero since they determine the quadrature component which does not influence the value of the metric anyway. Therefore, the complexity is cut down to $O(\sqrt{M})$ instead of $O(M)$. Likewise, for the last $B/2$ bits we define:

$$\underset{\frac{B}{2}+1 \leq i \leq B}{m_3(k,i)} = -\frac{|h_k|^2}{\sigma^2} \min_{\substack{d_k^j \in \{0,1\}, \frac{B}{2}+1 \leq j \leq B, j \neq i \\ d_k^j = 0 \text{ if } 1 \leq j \leq \frac{B}{2}}} |\Im(z_k) - \Im(s_k)|^2 \quad (12)$$

Finally, we tried to derive a recursive formula $m_4(k,i)$ for the computation of bit metrics in Gray-mapped M-QAM, as suggested in [4] and [5]. We considered symbol ML metrics, for which the metric is obviously the euclidean distance:

$$\hat{s} = \underset{\mathbf{s}\in\mathcal{S}}{\operatorname{argmax}} -\sum_k |y_k - h_k s_k|^2 \quad (13)$$

$$= \underset{\mathbf{s}\in\mathcal{S}}{\operatorname{argmax}} \sum_k \underbrace{-|h_k|^2|s_k|^2 + 2\Re(y_k h_k^* s_k^*)}_{\hat{=} m(k)} \quad (14)$$

(15)

We illustrate the computation for 16-QAM, but the extension to higher order constellations is straightforward. In the 16-QAM case, equations (9) and (10) result in:

$$\Re(s) = \frac{1}{\sqrt{10}} u_1(2-u_2) \quad (16)$$

$$\Im(s) = \frac{1}{\sqrt{10}} u_3(2-u_4) \quad (17)$$

First, $m(k)$ can be expressed as the sum of $m(u_1,u_2,k)$ and $m(u_3,u_4,k)$ defined by:

$$m(u_1,u_2,k) = -|h_k|^2\Re(s_k)^2 + 2\Re(y_k h_k^*)\Re(s_k) \quad (18)$$

$$m(u_3,u_4,k) = -|h_k|^2\Im(s_k)^2 + 2\Im(y_k h_k^*)\Im(s_k) \quad (19)$$

We now detail the generation of bit metrics for u_1 and u_2 from $m(u_1,u_2,k)$. First, we remove constant terms which do not influence Viterbi decoding:

$$m(u_1,u_2,k) = -\frac{|h_k|^2}{10} u_1^2(4+u_2^2-4u_2) + \frac{2\Re(y_k h_k^*)}{\sqrt{10}} u_1(2-u_2)$$

$$= -\frac{4|h_k|^2}{10} u_2 + \frac{2}{\sqrt{10}} \Re(y_k h_k^*) u_1(2-u_2) \quad (20)$$

where the second equality holds up to an additive constant. We decide to compute the conditional expectation, assuming that no a priori knowledge on the information bits probability is available apart from the fact that they are i.i.d. with $p(0) = p(1) = \frac{1}{2}$, (as assumed in the first iteration of Turbo demodulation schemes [9]):

$$m(u_1,k) \hat{=} \mathrm{E}\left[m(u_1,u_2,k)|u_1\right] \quad (21)$$

$$= \frac{m(u_1,u_2=+1,k)}{2} + \frac{m(u_1,u_2=-1,k)}{2} \quad (22)$$

$$m(u_2,k) \hat{=} \mathrm{E}\left[m(u_1,u_2,k)|u_2\right] \quad (23)$$

$$= \frac{m(u_1=+1,u_2,k)}{2} + \frac{m(u_1=-1,u_2,k)}{2} \quad (24)$$

Equation (22) leads to:

$$m(u_1,k) = \frac{4}{\sqrt{10}} \Re(y_k h_k^*) u_1 \quad (25)$$

In order to obtain a recursive expression, we insert $m(u_1,k)$ into $m(u_1,u_2,k)$ prior to computing (24). However, we first want to remove the dependence on u_1 from the second term of $m(u_1,u_2,k)$ in (20) since otherwise it would be cancelled during the expectation calculation. Therefore in the derivation of $m(u_2,k)$, we make the assumption that bit u_1 is reliable enough so that:

$$u_1\Re(y_k h_k^*) = |u_1\Re(y_k h_k^*)| = \frac{\sqrt{10}}{4}|m(u_1,k)| \quad (26)$$

Note that such an assumption would be more realistic with Ungerboeck's constellations for which some bits are much more protected than others.

The second metric $m(u_2,k)$ can now be computed by inserting (26) into (24), and by scaling both metrics $m(u_1,k)$ and $m(u_2,k)$ by the same factor $\frac{\sqrt{10}}{4}$. Finally, we can define a fourth Viterbi metric $m_4(k,i,M)$ which has an expression dependent on the QAM order M:

$$M = 16$$

$$m_4(k,1,M) = \Re(y_k h_k^*) u_k^1 \quad (27)$$

$$m_4(k,2,M) = -\frac{|h_k|^2}{\sqrt{10}} + \frac{1}{2}|m_4(k,1,M)| \quad (28)$$

$$m_4(k,3,M) = \Im(y_k h_k^*) u_k^3 \quad (29)$$

$$m_4(k,4,M) = -\frac{|h_k|^2}{\sqrt{10}} + \frac{1}{2}|m_4(k,3,M)| \quad (30)$$

Likewise, bit metrics can be obtained for the 64 QAM:

$$M = 64$$

$$m_4(k,1,M) = \Re(y_k h_k^*) u_k^1 \quad (31)$$

$$m_4(k,2,M) = -\frac{2|h_k|^2}{\sqrt{42}} + \frac{1}{2}|m_4(k,1,M)| \quad (32)$$

$$m_4(k,3,M) = -\frac{|h_k|^2}{2\sqrt{42}} + \frac{1}{2}|m_4(k,2,M)| \quad (33)$$

$$m_4(k,4,M) = \Im(y_k h_k^*) u_k^4 \quad (34)$$

$$m_4(k,5,M) = -\frac{2|h_k|^2}{\sqrt{42}} + \frac{1}{2}|m_4(k,4,M)| \quad (35)$$

$$m_4(k,6,M) = -\frac{|h_k|^2}{2\sqrt{42}} + \frac{1}{2}|m_4(k,5,M)| \quad (36)$$

The metric m_4 has a complexity in $\mathcal{O}(log_2(M))$, which can make it attractive for high order QAMs.

III. PERFORMANCE COMPARISON

We compared the performance of Viterbi decoding with metrics m_1, m_2, m_3 and m_4 in the context of the IEEE 802.11a physical layer on AWGN channel and on ETSI BRAN-A typical indoor channel model (50 ns r.m.s. delay spread). Perfect synchronization and channel estimation are assumed. The packet size was set to 128 bytes and the PDU error rate is plotted on figures 3 and 4. Three physical modes were simulated: (QPSK, code rate 1/2), (16QAM,code rate 1/2) and (64QAM,code rate 3/4).

As expected, the performance of all metrics is the same for trivial QAM (BPSK and QPSK). The log-sum approximation does not significantly deteriorate the PER performance even for 64 QAM. The recursive metrics degrade the performance by less than half a dB for 16 QAM, and less than 1dB for 64 QAM.

IV. Conclusion

We investigated in the context of an OFDM transmission the impact of well-known approximations for the derivation of Viterbi bit metrics, as well as a recursive formula specific to Gray mapped QAM. A significant reduction of computational complexity can be obtained by simple approximations which do not impact the packet error rate performance, while the least complex scheme exhibits a 1dB loss for 64QAM. Which metric is best for implementation depends on the complexity/performance trade-off of the system, and this paper provides results enabling implementers to make a choice.

Note

This work was performed as part of the IST Broadway Project [7], for which very high speed Viterbi decoding of high order QAM BICM-OFDM transmissions was considered.

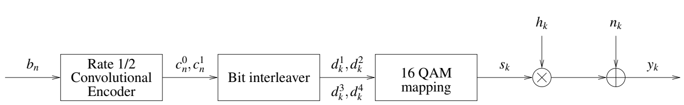

Fig. 1. Transmitter and channel model illustrated in QAM-16 code rate 1/2 case

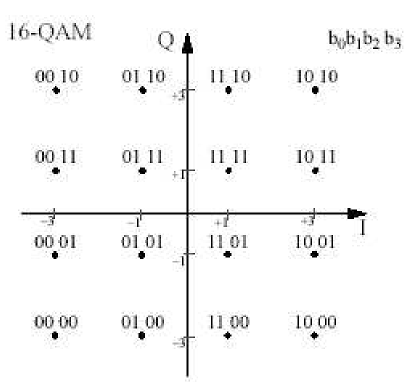

Fig. 2. Gray-mapped 16 QAM constellation

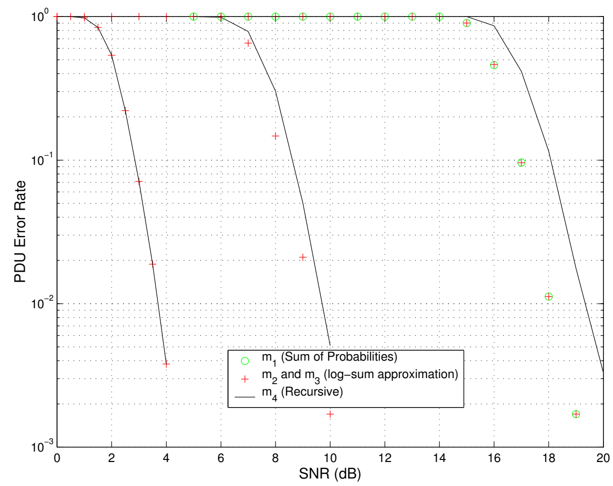

Fig. 3. PER performance of Viterbi decoding with various bit metrics (IEEE802.11a, AWGN channel)

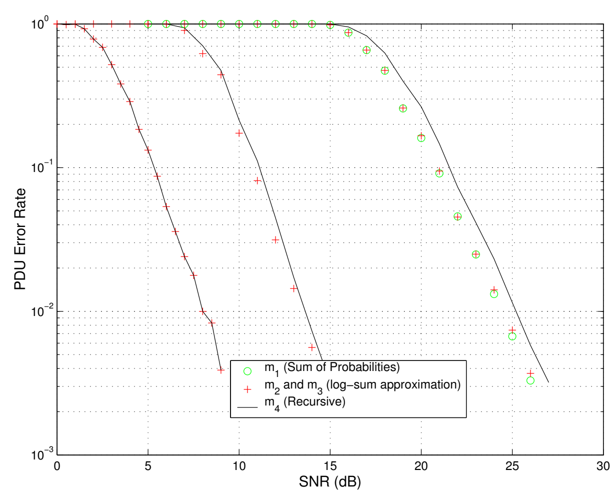

Fig. 4. PER performance of Viterbi decoding with various bit metrics (IEEE802.11a, BRAN-A channel)

REFERENCES

[1] E. Akay and E. Ayanoglu. Low Complexity Decoding of Bit-Interleaved Coded Modulation for M-ary QAM. In *International Communications Conference*, Paris, France, June 2004.

[2] M. Alard and R. Lassale. Principles of Modulation and Channel Coding for Digital Broadcasting for Mobile Receivers. *European Broadcasting Union Review Technical*, 224:168–190, August 1987.

[3] G. Caire, G. Taricco, and E. Biglieri. Bit-Interleaved Coded Modulation. *IEEE Trans. on Information Theory*, 44:927–946, May 1998.

[4] P Vila et al. Reduced Complexity M-ary binary decoders for turbo-equalization. In *International Symposium on on Turbo Codes and Related Topics*, Brest, France, September 2000.

[5] Wei Fu, Jian Liu, and Guangxi Zhu. Bit-wise demodulation for 16-QAM with BICM. In *International Conference on Circuits and Systems for Communications*, pages 444–449, June 2002.

[6] IEEE 802.11a. Part 11:Wireless LAN Medium Access Control (MAC) and Physical Layer specifications (PHY) - High Speed Physical Layer in the 5GHz band. IEEE Std 802.11a-1999, IEEE Standards Department, New York, January 1999.

[7] IST Broadway Project. http://www.ist-broadway.org.

[8] T. May, H. Rohling, and V. Engels. Performance Analysis of Viterbi Decoding for 64-DAPSK and 64-QAM Modulated OFDM Signals. *IEEE Trans. on Communications*, 46(2):182–190, February 1998.

[9] Bertrand Muquet, Marc de Courville, Pierre Duhamel, and Georgios Giannakis. Turbo Demodulation of Zero-Padded OFDM Transmissions. *IEEE Trans. on Communications*, 50(11), November 2002.