BER and PER estimation based on Soft Output decoding

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ABSTRACT

***Abstract*— A novel PER estimation method based on the Log Likelihood Ratios of a soft output decoder is proposed. Classically, PER estimation based on soft output decoding depends on an estimate of the SNR. The contribution of this paper is a PER estimate that is not biased by such SNR estimation uncertainty. We compare the proposed estimation method with the estimator proposed by Hoher first by analytic expression of the estimator's bias for uncoded BPSK modulation transmitted over an AWGN channel. The study is extended to OFDM coded transmission by simulation means.**

I. INTRODUCTION

There is a trend in current communication systems to optimize the transmission parameters to the varying link quality. Actually, in wireless links (e.g. WLANs, 3G), the quality can fluctuate due to fading and interference, while in wired systems (e.g. xDSL) cross-talk is responsible for link quality variations. This results in a variation of the short-term Shannon Capacity of the channel. The communication system has to permanently adapt its bit rate by selecting the optimum constellation and coding rate. These adaptive mechanisms can be either Adaptive Modulation and Coding (AMC) or Hybrid Automatic Repeat reQuest (HARQ), and aim at maximizing the momentary throughput of the link, possibly under some QoS constraints, given a metric which is supposed to determine the channel quality. The momentary Signal to Noise Ratio (SNR) is often the adopted metric, however this metric is not optimum because it does not fully determine the throughput. It has been shown that system performance can be significantly improved if the AMC scheme is based on a more detailed description of the channel state. In [2] methods for selecting the PHY mode based on the estimated channel transfer function are presented and compared to conventional algorithms. Since the PER fully determines the throughput, another solution can be to try to estimate it by counting the erroneous packets. The problem is that the estimate of the PER obtained with this method is very inaccurate or else many packets are required which prevents for fast adaptation to channel conditions. The required observation interval can correspond to many times the channel coherence time and yields very sub-optimal performance (in [3], we observed a loss greater than 2 dB).

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When the receiver implements soft output decoding (e.g. the forward-backward algorithm [1]), a BER estimator can exploit the soft information on decoded bits. BER estimators based on the Log Likelihood Ratios (LLR) at the decoder output were presented by Loeliger [5] and further analyzed by Land and Hoeher [6]. Likewise, in [7], a BER estimator is proposed based on the statistical moments of the LLR distribution. All those BER estimators implicitly assume that the signal to noise ratio is known to the decoder. A perfect knowledge of the SNR is however hard to achieve due to measurement errors or channel gain variations. Although such a knowledge is not necessary for correct decoding, since soft-output decoders tolerate a significant SNR estimation error without BER degradation ([8]), the LLR distribution strongly depends on the accuracy of the SNR estimate. In this paper, we introduce a new BER estimator which is also based on the LLR distribution, but which does not exhibit a dependence on the SNR uncertainty δ.

We quantify the impact of SNR estimation error on LLR based estimators by analytical calculations for uncoded transmission over AWGN channel, and by computer simulations for convolutionally coded transmission in an OFDM system. All our analytic results refer to an uncoded BPSK modulation transmitted over an AWGN channel.

A simple uncoded system model is considered in both sections II & III, in order to introduce the notations and do the analytical study. The obtained results will further be applied and validated on a coded OFDM transmission in section IV.

II. Theoretical Background

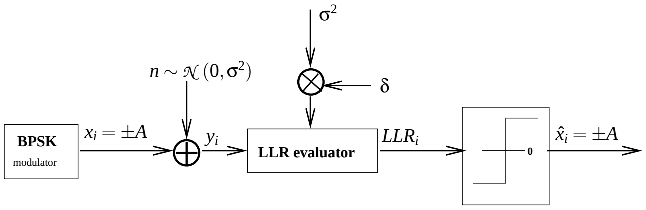

Fig. 1. Transmission path model.

Consider the transmission path of Fig. 1, let $x_i = \pm A$ be the transmitted symbol, n_i the Additive White Gaussian Noise (AWGN) of variance $\sigma^2 = \frac{N_0}{2}$, the scaling factor δ is the relative estimation error on σ^2 and y_i the received signal. It can be verified that LLR_i can be modeled as i.i.d. random variable following the distribution (1) related to the i-th transmitted symbol x_i of the packet; if the coded bits are modulated using a BPSK or QPSK modulation over an AWGN channel, then the probability distribution of LLR_i is given by:

$$p_{LLR} = p[x=-A]\mathcal{N}(-\frac{\alpha}{\delta},\frac{\beta^2}{\delta^2}) + p[x=+A]\mathcal{N}(\frac{\alpha}{\delta},\frac{\beta^2}{\delta^2}) \qquad (1)$$

where $\mathcal{N}(m,\sigma^2)$ denotes the Normal distribution of mean m and variance σ^2 and $\beta^2 = 2\alpha = \dfrac{4A^2}{\sigma^2}$. Moreover, under the above hypothesis, the BER of the uncoded transmission is equal to $Q\left(\frac{A}{\sigma}\right)$ (with $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{u^2}{2}}du$). Therefore the BER is univocally identified by the ratio $A/\sigma = \alpha/\beta$. Unfortunately, α and β are not obviously estimated from p_{LLR}, since the two tails of the Gaussian distributions overlap. On Fig. 2 it is shown how the the probability distribution of LLR_i depends on δ at fixed SNR. Furthermore, while the statistical moments of p_{LLR} depend on both $\frac{A}{\sigma}$ and δ, the BER does not. For convolutionally and turbo coded transmission, recent studies [9], [8], [10] investigate the sensitivity of soft output decoders (Forward-Backward, MAP, MAX-Log-MAP, etc) to δ. These decoders need an a priori knowledge of σ^2 to correctly estimate the a posteriori probability of the decoded bits [10]. In [9], [8] it is shown how the above decoders return an almost constant BER value as long as δ is in the range [-2 dB, 6 dB] assuming a BPSK transmission over an ideal AWGN channel. Eventually:

$$BER(\frac{A}{\sigma},\delta) \simeq BER(\frac{A}{\sigma},1) \qquad (2)$$

Unfortunately, $\frac{A}{\sigma}$ is unknown.

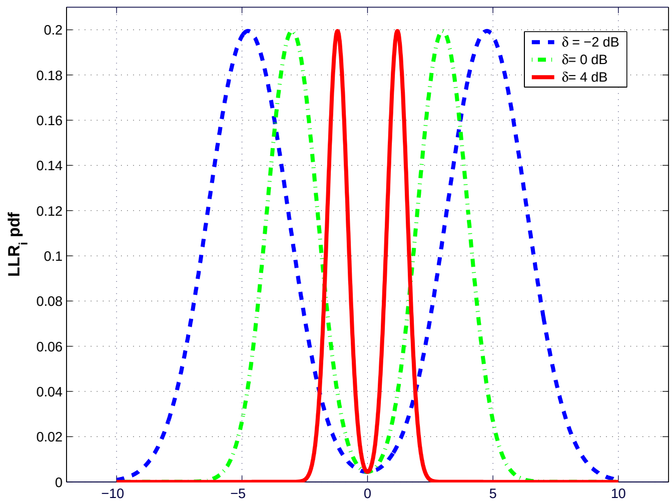

Fig. 2. Probability distribution of LLR_i for $\delta = \{-2,0,4\}$ dB

III. BER estimation

In this section, we analyze two BER estimation methods. We show that the first method, already known in the literature, depends on SNR uncertainty δ. Then, we propose a new method, Method 2, insensitive to δ.

A. Method 1

Considering the transmission path of Fig. 1, let N be the packet length, a BER estimator (proposed by Hoeher and al. in [6]) is:

$$\hat{BER}_1 = \frac{1}{N}\cdot\sum_{i=1}^{N}\frac{1}{1+e^{\,|LLR_i|}} \qquad (3)$$

Note that since $APP(+A) = \dfrac{1}{1+e^{-LLR}}$ and $APP(-A) = \dfrac{1}{1+e^{LLR}}$, then (3) can be equivalently formalized as:

$$\hat{BER}_1 = \frac{1}{N}\cdot\sum_{i=1}^{N} min(APP_i(-A), APP_i(+A)) \qquad (4)$$

We compute the $E[\hat{BER}_1]$ (where $\hat{BER}_1$ is given by formula (3)) for an uncoded BPSK or QPSK modulation in order to study the bias of the estimator. Contrary to prior art we consider the presence of an estimation error δ on the channel noise variance. For simplification, we provide the demonstration only for a BPSK modulation:

$$
\begin{aligned}
&E[B\hat{E}R_1] \\
&= E[\frac{1}{1+e^{|LLR|}}] = \int_{-\infty}^{+\infty} \frac{1}{1+e^{|t|}} \cdot p_{LLR}(t)dt \\
&= \frac{1}{2} \cdot \Bigg[\underbrace{\int_{-\infty}^{0} \frac{1}{1+e^{-t}} \cdot \mathcal{N}(\frac{\alpha}{\delta}, \frac{\beta^2}{\delta^2})dt}_{BER-I_2} + \underbrace{\int_{0}^{\infty} \frac{1}{1+e^{t}} \cdot \mathcal{N}(\frac{-\alpha}{\delta}, \frac{\beta^2}{\delta^2})dt}_{BER-I_2} \\
&\quad + \underbrace{\int_{-\infty}^{0} \frac{1}{1+e^{-t}} \cdot \mathcal{N}(\frac{-\alpha}{\delta}, \frac{\beta^2}{\delta^2})dt}_{I_1} + \underbrace{\int_{0}^{\infty} \frac{1}{1+e^{t}} \cdot \mathcal{N}(\frac{\alpha}{\delta}, \frac{\beta^2}{\delta^2})dt}_{I_1} \Bigg] \\
&= \quad BER + I_1 - I_2
\end{aligned} \tag{5}
$$

where:

$$
I_1 = \int_0^{\infty} \frac{1}{1+e^t} \cdot \frac{\delta}{\beta \cdot \sqrt{2\pi}} \cdot e^{-\frac{(t-\frac{\alpha}{\delta})^2}{2\frac{\beta^2}{\delta^2}}} dt
$$

and

$$
I_2 = \int_0^{\infty} \frac{e^t}{1+e^t} \cdot \frac{\delta}{\beta \cdot \sqrt{2\pi}} \cdot e^{-\frac{(t+\frac{\alpha}{\delta})^2}{2\frac{\beta^2}{\delta^2}}} dt
$$

Finally, the bias of the estimator can be expressed as:

$$
I_1 - I_2 = \frac{\delta}{\beta \cdot \sqrt{2\pi}} \cdot \int_0^{\infty} \frac{e^{-\frac{(t^2\delta^2+\alpha^2)}{2\beta^2}}}{1+e^t} \cdot \left[e^{\frac{\delta t}{2}} - e^{(1-\frac{\delta}{2})t} \right] \cdot dt
$$

then, $I_1 - I_2 \geq 0$ if $t(\delta - 1) \geq 0$. Indeed, since $t \in [0, +\infty]$:

$$
\begin{cases}
1 > \delta > 0 & \Longrightarrow I_1 - I_2 < 0 \quad \Longrightarrow B\hat{E}R_1 < BER \\
\delta = 1 & \Longrightarrow I_1 - I_2 = 0 \quad \Longrightarrow B\hat{E}R_1 = BER \\
\delta > 1 & \Longrightarrow I_1 - I_2 > 0 \quad \Longrightarrow B\hat{E}R_1 > BER
\end{cases} \tag{6}
$$

Consequently, $B\hat{E}R_1$ is unbiased for uncoded BPSK or QPSK modulations only if $\delta = 1$. Simulation results (see section IV) will qualitatively extend the validity of this result for Convolutionally coded OFDM transmission.

B. Method 2

Considering the same transmission path as section III-A, we propose a new BER estimator that can be obtained at each packet observation by computing the mean and standard deviation of $|LLR|$ from the observation of the LLR_i. Then, the proposed estimator is defined as:

$$
B\hat{E}R_2 = h(\Lambda) \tag{7}
$$

where h is a function that univocally links the BER estimate to the link quality metric Λ defined as:

$$
\Lambda = \frac{E[\,|LLR|\,]}{\sqrt{E[(\,|LLR| - E[\,|LLR|\,])^2]}} \tag{8}
$$

From the LLR distribution provided in (1), (8) can be expressed as (algebraic manipulations are omitted):

$$
\Lambda = \frac{\sqrt{2} \cdot \left(-\frac{1}{2} + Q\left(\frac{\alpha}{\beta}\right) - \frac{1}{\sqrt{2\pi}\cdot\frac{\alpha}{\beta}} \cdot e^{-\frac{1}{2}\cdot(\frac{\alpha}{\beta})^2} \right)}{\sqrt{\left(\frac{1+\left(\frac{\beta}{\alpha}\right)^2}{2} \right) - 2 \cdot \left(-\frac{1}{2} + Q\left(\frac{\alpha}{\beta}\right) - \frac{1}{\sqrt{2\pi}\cdot\frac{\alpha}{\beta}} \cdot e^{-\frac{1}{2}\cdot(\frac{\alpha}{\beta})^2} \right)^2}} \tag{9}
$$

Studying the expression (9) we infer that Λ is monotonically decreasing depending only on the ratio $\frac{\alpha}{\beta}$. Consequently, so does $B\hat{E}R_2$, being independent from δ. The function h can be implemented by means of a Look Up Table (LUT) which can be easily obtained analytically since both Λ and BER are functions of the same ratio $\frac{\alpha}{\beta}$. The LUT of (7) can be easily computed from (9), since $\frac{\alpha}{\beta} = \frac{A}{\sigma}$ and $BER = Q\left(\frac{A}{\sigma}\right)$. Mean and variance of $|LLR|$ over the N observed symbols can be computed either directly or by first computing a histogram of the LLR observed values. The second approach results typically in a smoother estimation, and it may return better results for the estimation of mean and variance, but this is not the scope of this letter. Moreover, for small values of BER, the impact of the overlapping of the two normal distributions in (1) becomes negligible, and we have $\Lambda \approx \frac{\alpha}{\beta}$. Such a situation corresponds to a typical operational context since the target BER of a system is often below 10^{-4}.

IV. Simulation Results

We start by studying the sensitivity of soft output decoders to δ in the case of QAM modulated OFDM transmission. We observed by means of simulation the sensitivity of a 16-state BCJR [1] to δ. On table I are summarized our simulation results gained for a BPSK, QPSK, 16-QAM and 64-QAM modulation. No puncturing was applied to the convolutional

Modulation	$\mathcal{A} \in$ **[dB]**
64-QAM	[-2,9]
16-QAM	[-1,11]
QPSK	[-2,10]
BPSK	[-3,8]

TABLE I

VALIDITY OF THE APPROXIMATION (2)

code. On Fig. 3 and 4 we show as an example the curves of BER versus $\frac{E_s}{N_0}$ varying $\delta \in [-2, 7]$ dB for a convolutionally coded BPSK and 64-QAM modulation.

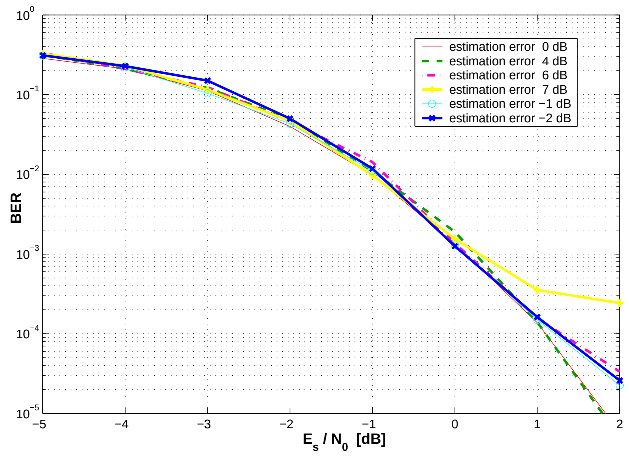

Fig. 3. BER versus $\frac{E_s}{N_0}$ for BPSK modulation with an estimation error in the range of [-2,7] dB

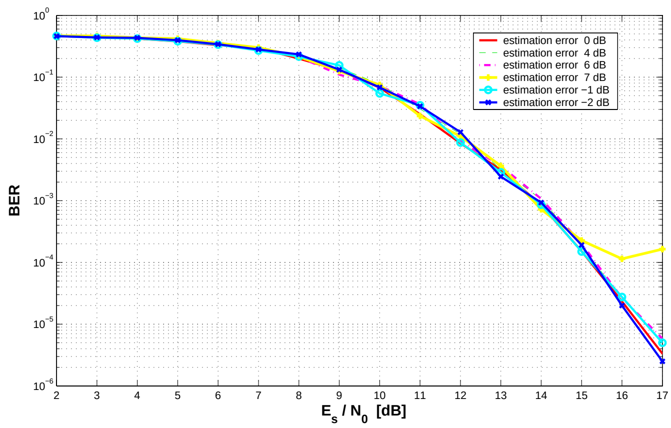

Fig. 4. BER versus $\frac{E_s}{N_0}$ for 64-QAM modulation with an estimation error in the range of [-2,7] dB

Eventually, if $\delta \in \mathcal{A}$, formula (5) holds also for QAM modulated OFDM transmission. Then, if $\delta \in \mathcal{A}$, a BER estimate based on soft output decoding is consistent. In this section the two BER estimation methods are compared in terms of BER versus SNR. All following reported results are related to the (64QAM, code 3/4) mode of the IEEE 802.11a/g convolutionally coded OFDM system. The soft-output decoder is a BCJR. A LUT is used which associates the PER for a given packet size with the BER. In order to apply the two BER estimators at the decoder output, we assumed a Gaussian-like distribution of the a posteriori log-ratios [4]. Simulations show that this assumption remains good when the channel is flat (no multipath): both estimators are unbiased at $\delta = 1$ but estimator 1 gets severely biased for values of δ which do not significantly degrade the BER performance (e.g. $\delta = 2dB$). However, the Gaussian assumption for the LLR becomes erroneous in most Wireless LAN channels. Table 1 compares the standard deviation of the bias of both PER estimators over 100 realizations of a typical indoor (ETSI BRAN-A) channel at various operating points. It can be verified that when $\delta = 1$, estimator 1 is still reliable, whereas estimator 2 is severely biased, because it relies on the erroneous Gaussian LLR distribution assumption. When δ varies uniformly over the $[-2;6]$ dB range, estimator 2 resists better for the reasons mentioned previously, but both estimators exhibit significant bias anyway.

PER reference	$\delta = 0$ **dB** Estimator 1	Estimator 2	$\delta \in [-2;6]$ **dB** Estimator 1	Estimator 2
0.20	0.0180	0.1048	0.1662	0.1072
0.15	0.0139	0.0835	0.1228	0.0756
0.10	0.0089	0.0586	0.0807	0.0484
0.05	0.0044	0.0187	0.0406	0.0331

TABLE II

STANDARD DEVIATION OF PER ESTIMATOR BIAS

V. CONCLUSIONS AND FURTHER WORK

PER estimation is a key element for adaptive mechanisms such as AMC and HARQ. Packet by packet PER estimation is a desired feature for AMC. For instance, having an updated estimate of the channel quality at its disposal, the adaptive mechanism can easier track the fast channel variation avoiding or limiting a catastrophic link adaptation. If a soft output decoding is employed, PER estimation can be obtained at each packet decoding making use of both estimation methods (3) and (8). In this paper we have shown how the Hoeher estimator (5) returns an unbiased estimate if a perfect SNR estimation is available. Unfortunately the SNR is estimated with an uncertainty. In such case, the proposed estimation method (8) results in a better PER estimate since is not biased. Anyway, both estimators present dependence on the particular channel characteristics in frequency selective block fading channels. It could be interesting to compare such estimation methods with a PER estimator that uses an estimate of the channel characteristics.

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