A new method for Joint Cancellation of Clock and Carrier Frequency Offsets in OFDM receivers over Frequency Selective Channels

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Abstract - In this paper, a new estimator is presented which enables a baseband digital compensation of carrier and clock frequency offsets in OFDM systems. The estimator is obtained by applying the joint maximum likelihood criterion to one or several frequency domain symbols, with a model that explicitely takes into account the channel frequency selectivity. Under certain assumptions a linearization can be applied, leading to an analytical expression of the estimator. Otherwise, two-dimensional optimization is required. Several versions of the estimator are presented according to whether the modulation is PSK or QAM, differential or coherent. The performance improvement is evaluated in two contexts: Digital Audio Broadcasting and Wireless Local Area Networks (HIPERLAN/2-like systems). In the context of vehicular DAB, the influence of Doppler is emphasized. In HIPERLAN/2, simulations show that a significant gain (up to 1 dB) can be achieved with the new method compared to basic Linear Least Squares estimators, enabling an all-digital compensation of frequency offsets with negligible degradation of the BER performance.

I. Introduction

Though unnoticed for some time, there has been an increasing interest towards multicarrier and in particular Orthogonal Frequency Division Multiplexing (OFDM) [1], not only for digital audio- and video-broadcasting (DAB and DVB) but also for high-speed modems over Digital Subscriber Line (xDSL), and more recently for local area wireless broadband systems (ETSI BRAN HiperLAN/2 (HL2) similar to IEEE802.11a and ARIB 5GHz MMAC).

An asset of OFDM is that, when associated with coding and interleaving, it shows high resistance against multipath propagation and enables a very simple equalization scheme. However it is also very sensitive to carrier and clock Frequency Offsets (FO), which generate Inter Carrier Interference (ICI) and parasitic rotations of the subcarriers [5, 6]. Several techniques already exist for estimating and canceling a carrier FO based on time correlations (e.g: [7]). A frequency domain estimator of the carrier and clock FO is also proposed in [3] based on Linear Least Squares (LLS) fitting. Unfortunately, this approach assumes that the channel attenuation is the same for all subcarriers, which leads to a performance loss.

This paper presents a new Joint Maximum Likelihood (ML) frequency domain estimator of the carrier and clock FO relying on a model that explicitly takes into account the channel frequency selectivity. Since the proposed method processes frequency domain received symbols, as in [3], it is assumed in the following that the carrier frequency offset is preliminary reduced down to typically a few percents of the inter-carrier spacing so that the signal is not totally corrupted by ICI. Such a compensation can be performed by a low arithmetical complexity algorithm such as the one provided in [4]. Thus the joint estimation technique detailed in this paper enables a refined compensation of the residual carrier frequency error and the suppression of the clock frequency offset. It is derived in two contexts: i) differential phase modulations (Digital Audio Broadcasting case), either Data Aided (DA) or not ii) coherent modulations, pilot symbols assisted (DA) in the HL2 context. For small rotations and high Signal to Noise Ratio (SNR), a linearization of the estimator can be applied. In this case, an extension to wireless OFDM of the timing estimator described in [8] is obtained.

The BER performance of the new synchronization algorithm is assessed by simulations in two contexts. In DAB, the LLS estimator proves to be accurate enough to mitigate the degradation resulting from parasitic rotations. We also show that FO compensation reduces the effects of Doppler. In HL2, the new method outperforms LLS estimator by up to 1 dB. A simplification of our ML estimator is detailed for systems like HL2 that enjoy linked sources (same oscillator for downconversion and sampling). Finally, the joint ML estimator enables a full digital synchronization of coherent receivers even without the linked sources feature.

The paper is organized as follows. Section II presents a modeling of FO in OFDM systems. The estimator is derived in section III. Simulations are finally provided in section IV.

II. Modeling of the OFDM signal with frequency offsets

This section presents a discrete model of the OFDM system which takes into account channel attenuation, noise, carrier and clock FO impairments. The OFDM transmitter is detailed figure 1.

By construction, the incoming high rate symbol stream is mapped onto K sub-carriers modulated at a lower rate. The lth OFDM symbol of the frame has K components and is denoted by $\mathbf{S(l)} = (S_{-\frac{K}{2}}(l), \cdots, S_{-1}(l), S_1(l), \cdots, S_{\frac{K}{2}}(l))$. $S_m(l)$ can be either a QAM or a PSK constellation symbol, and is transmitted on subcarrier of frequency $F_c + \frac{m}{NT}$ where $N > K$ is the size of the Inverse Fast Fourier Transform (IFFT) modulator. The first and central IFFT inputs fed with $N-K$ zeros correspond to producing an oversampled version of the time domain signal. At the IFFT output, a guard interval of E samples is inserted at the beginning of each block, and consists of a cyclic extension of the time domain OFDM symbol. After Parallel to Serial (P/S) and Digital to Analog Conversion (DAC), the signal is up-converted and transmitted over the mobile radio channel. At the receiver, symmetrical operations are performed: down-conversion, Analog to Digital Conversion (ADC), suppression of the guard interval and finally FFT demodulation. It is well known [1] that, as long as the cyclic prefix is larger than the channel impulse response, the effect of the channel on the mth FFT output just results in a complex gain denoted here by $H_m(l)$. Therefore, under perfect synchronization assumption, the block output of the demodulator is $\mathbf{R(l)}$ of components:

$$R_m(l) = H_m(l)S_m(l) + b_m(l), \ -\frac{K}{2} \leq m \leq \frac{K}{2}, m \neq 0 \quad (1)$$

where $b_m(l)$ denotes the Additive White Gaussian Noise (AWGN). As already stated in the introduction and discussed later in this section, the ICI will be neglected since a precompensation of the FO is assumed. In that case only parasitic rotations remain in the frequency domain. In equation (2), one can verify that this rotation can be split into two parts: the first one refered to as Common Phase Error (CPE) is proportional to the carrier FO and constant for all subcarriers. The second one is proportional to the clock FO and the subcarrier frequency. Therefore we will call it: the Variable Phase Error (VPE). In the following, parameter A is refered to as the CPE and parameter B as the VPE for simplicity.

$$R_m(l) = H_m(l)S_m(l)e^{jl(A+mB)} + b_m(l) \quad (2)$$

Denoting by $f_e \triangleq NT(\Delta F_c)$ the carrier FO normalized by the subcarrier spacing and $-\frac{\Delta T}{T}$ the relative clock FO, resulting in the following expression for A and B:

$$A = 2\pi\frac{N+E}{N}f_e, \ B = 2\pi\frac{N+E}{N}\frac{\Delta T}{T} \quad (3)$$

Before trying to compensate phase rotations, as already mentioned, the ICI must remain under a certain threshold. For instance in HL2, the BER degradation caused by ICI is less than 0.1 dB for SNR up to 30 dB if: i) $|f_e| < 2\%$ at SNR=10 dB and $|f_e| < 0.3\%$ at SNR=30 dB ii) $|\frac{\Delta T}{T}| < 50$ppm. In practice, this always requires a precompensation of the carrier FO in the time domain (before applying the FFT), whereas clock correction is not necessary. Because they are not affected by ICI, time domain estimators [2, 7] are well suited to this pre-compensation. Some very specific frequency domain estimators [4] can also be used with this aim. However, the CPE and VPE still produce a phase drift that grows as l increases. In order to quickly evaluate the impact of CPE and VPE on the BER performance, the rotations are assimilated to a distortion of power D, and the resulting Signal to Distortion Ratio (SDR) is derived. In section IV simulations validate the fact that BER is a function of the Signal to Distortion plus Noise Ratio (SDNR). Therefore, the SDR must be much higher than the SNR otherwise a degradation of the BER performance occurs. The SDR associated to CPE and VPE are respectively $(SDR)_{\text{CPE}}(l) = \frac{1}{|1-e^{jlA}|^2} = \frac{1}{4\sin(\frac{lA}{2})^2}$ and $(SDR)_{\text{VPE}}(l) = \frac{1}{K}\sum_{m=-\frac{K}{2}, m\neq 0}^{\frac{K}{2}} 4\sin\left(\frac{lmB}{2}\right)^2$. When both are present in a system, the distortions are added. On figure 2, the SDR is plotted for various instants of the frame in the context of HIPERLAN/2. In a typical office environment, the required SNR ranges from 10 dB to 30 dB according to the constellation size selected. However, at SNR=10 dB, with $f_e = 2\%$, the SDR is only 10 dB for the first symbol, which implies a 3 dB degradation of the BER performance. Therefore, CPE tracking is compulsory in HL2. Moreover, since the standard specifies a transmitter and receiver oscillator stability of +/- 20 ppm, a 40 ppm clock FO can be reached. Thus VPE tracking is a must even when considering short frames.

III. Derivation of the carrier and clock estimators

In this section, various carrier and clock FO estimators are derived among which the new joint ML one.

III.1. Joint Maximum Likelihood estimator

Let the ML criterion be applied to the observations $R_m(l)$ provided by equation (2). Assuming that the channel as well as the transmitted data are known, the ML estimates for the CPE and the VPE are thus given by:

$$(\hat{A}, \hat{B}) = \text{ArgMax}_{(x,y)} \log[p((\mathbf{R}(l_i))_{1\leq i\leq L} \mid A = x, B = y, (\mathbf{H}(l_i))_{1\leq i\leq L}, (\mathbf{S}(l_i))_{1\leq i\leq L})] \quad (4)$$

where L stands for the number of OFDM symbols taken into account in the estimation process. Since the noise is assumed AWGN, the expression $\sum_{i=1}^{L} \|\mathbf{R}(l_i) - \mathbf{T}(l_i)\|^2$ has to be minimized with $\mathbf{T}(l)$ defined as the vector of components $T_m(l) \triangleq H_m(l)S_m(l)e^{jl(x+my)}$. This expression can thus be developed as:

$$\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\left[\underbrace{|R_m(l_i)|^2+|T_m(l_i)|^2}_{\text{independant of }(x,y)}-2\Re\left(R_m(l_i)H_m^{\star}(l_i)S_m^{\star}(l_i)e^{-jl_i(x+my)}\right)\right]$$

where $(.)^{\star}$ stands for the conjugation operator. Denoting by $\mathbf{Z}(\mathbf{l})\triangleq\mathbf{R}(\mathbf{l})\mathbf{H}^{\star}(\mathbf{l})\mathbf{S}^{\star}(\mathbf{l})$, the estimates expression becomes:

$$(\hat{A},\hat{B})=\text{ArgMin}_{(x,y)}\left[-\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\Re\left(Z_m(l_i)e^{-jl_i(x+my)}\right)\right] \quad (5)$$

The first step of the estimation algorithm is the computation of $\mathbf{Z}(l)$ which basically removes the channel and the modulation phases. Then $\hat{A}$ and $\hat{B}$ can be obtained by classical two-dimensional minimization for solving equation (5) (e.g. applying Newton algorithm). Moreover, if A and B are small enough, equation (5) can be linearized using the Taylor series development of e^x and defining $F(x,y)$ as:

$$F(x,y)=\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\Re\left(R_m(l_i)H_m^{\star}(l_i)S_m^{\star}(l_i)\left(1-jl_i(x+my)\right)\right)$$

Thus the analytical expression of $\hat{A}$ and $\hat{B}$ is obtained by solving the following linear system:

$$\left\{\begin{array}{lll}\frac{\partial F}{\partial x}(\hat{A},\hat{B}) & = & 0\\ \frac{\partial F}{\partial y}(\hat{A},\hat{B}) & = & 0\end{array}\right.\Leftrightarrow\left\{\begin{array}{lll}\hat{A} & = & \frac{\alpha_1\alpha_2-\alpha_3\alpha_5}{\alpha_3\alpha_4-\alpha_2^2}\\ \hat{B} & = & \frac{-\alpha_2\hat{A}-\alpha_1}{\alpha_3}\end{array}\right. \quad (6)$$

where $\alpha_k=\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}} l_i\beta_k(m)\gamma_k\left(R_m(l_i)H_m^{\star}(l_i)S_m^{\star}(l_i)\right)$,

$\beta_1(m)=m;\ \beta_2(m)=-m;\ \beta_3(m)=-m^2;\ \beta_4(m)=-1;\ \beta_5(m)=1$

$\gamma_1(u)=\Im(u);\ \gamma_2(u)=\Re(u);\ \gamma_3(u)=\Re(u);\ \gamma_4(u)=\Re(u);\ \gamma_5(u)=\Im(u)$

III.2. Two Linear Least Squares estimators

In [3], the LLS estimation is performed directly on the phase rotations. Applying the same technique to the phase of $\mathbf{Z}(l)$ yields:

$$\left\{\begin{array}{lll}\hat{A} & = & \frac{1}{K\sum_{i=1}^{L}l_i^2}\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\left(l_i\angle Z_m(l_i)\right)\\ \hat{B} & = & \frac{1}{K\sum_{i=1}^{L}(ml_i)^2}\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\left(ml_i\angle Z_m(l_i)\right)\end{array}\right. \quad (7)$$

where $\angle$ returns the phase of its complex argument. Though this estimator is clearly suboptimum because the information on the amplitude of $\mathbf{Z}(l)$ is lost, it will serve as a reference for the simulations.

In the following we consider the case when $B=0$ (i.e the VPE is already compensated) and $L=1$. Then another LLS estimator can be obtained based on the rigourous model of equation (2) that is to say by taking into account the channel coefficients. Actually, with $\Phi(l)\triangleq e^{jlA}$, equation (2) turns into:

$$R_m(l)=H_m(l)S_m(l)\Phi(l)+b_m(l)$$

In this case both LLS and ML estimators provide the same result:

$$l\hat{A}=\angle\left(\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}Z_m(l)\right) \quad (8)$$

As highlighted in the following section, this estimator for the $B=0$, $L=1$ configuration is very convenient for implementation.

IV. Application to DAB and HIPERLAN/2

In this section, the estimators whose expression is given by equations (5), (6), (7) and (8) are evaluated in two separate contexts: the DAB system able to operate at vehicular speeds and HL2 restricted to pedestrian mobility. This way, in the following, various requirements will be derived and among the previously described estimators, the most suited ones for each context will be simulated.

IV.1. DAB context

A DAB receiver does not require any channel estimation since it is based on a Differential QPSK modulation of the data symbols $D_m(l)=S_m(l)S_m^{\star}(l-1)$. Actually the mth output of the differential demodulator is (noiseless case):

$$\begin{aligned}R_m(l)R_m^{\star}(l-1) &= H_m(l)H_m^{\star}(l-1)S_m(l)S_m^{\star}(l-1)e^{j(A+mB)}\\ &\approx |H_m(l)|^2D_m(l)e^{j(A+mB)}\end{aligned}$$

which inherently removes at the receiver the channel phase effects. Therefore, in DAB, equation (5) becomes:

$$(\hat{A,B})=\text{ArgMin}_{(x,y)}\left[-\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}}\Re\left(R_m(l)R_m^{\star}(l-1)D_m^{\star}(l)e^{-j(x+my)}\right)\right]$$

The previous equation leads to a Data Aided (DA) estimator, in which a decision $\hat{D}_m^{\star}(l)$ is taken on the differentially decoded symbol: $Z_m(l)\triangleq R_m(l)R_m^{\star}(l-1)\hat{D}_m^{\star}(l)$. A

Non Data Aided (NDA) estimator can be obtained by applying the folowing trick in order to remove the modulation effects without any slicer. It consists in multiplying the phase of the differentially decoded symbol by 4. Since $D_m(l) \in \left\{e^{j\frac{\pi}{4}}, e^{j\frac{3\pi}{4}}, e^{j\frac{5\pi}{4}}, e^{j\frac{7\pi}{4}}\right\}$, $4\angle D_m(l) = \pi$ the NDA estimator is thus:

$$(\hat{A},\hat{B}) = \frac{1}{4}\mathrm{ArgMin}_{(x,y)}\left[\sum_{i=1}^{L}\sum_{\substack{m\neq 0\\ m=-\frac{K}{2}}}^{\frac{K}{2}} \Re\left(\frac{(R_m(l)R_m^*(l-1))^4}{|R_m(l)R_m^*(l-1)|^3}e^{-j(x+my)}\right)\right]$$

Both the linearized DA and equation (7) LLS estimators are simulated in DAB mode 1: 300 MHz center frequency system using $K = 1536$ carriers over a typical static urban channel. The estimation is performed on the last $L = 4$ received symbols using a sliding window. An initial carrier frequency acquisition to within 3% of the subcarrier spacing is assumed, as well as a clock stability of 50 ppm. Results are presented figure 3. Although the linearized ML algorithm outperforms the LLS, it can be verified that the resulting difference in terms of BER is negligible. Note that, at high SNR, an irreducible error floor appears for ML explaining the better results of the LLS. This phenomenon comes from the linearization approximations and becomes predominant for the NDA estimator since parameters x and y are multiplied by 4.

In a mobile environment, both Doppler and FO generate parasitic rotations and, as a side effect, CPE and VPE tracking can partially compensate a Doppler spread. The rate at which the FO estimate is updated determines the bandwidth of the FO compensation loop which is approximately equal in our case to $\frac{1}{L(N+E)T}$. In order to benefit from a partial Doppler phase tracking, the correction loop bandwidth must be much larger than the Doppler bandwidth. Moreover, a comprehensive performance evaluation should take into account the phase noise which is also filtered by the loop.

Simulations have shown that either ML or LLS estimator can be considered in DAB, since they exhibit similar performance. In practice, if the oscillator accuracy is good enough (better than 20 ppm), only CPE tracking is required, and the estimator of equation (8) seems to be the best choice, since it is less complex (only one angle calculation is performed per OFDM symbol) and more efficient (not linearized). This conclusion cannot be extended to other systems, as confirmed in the next subsection.

IV.2. HIPERLAN/2 context

HL2 is a $K = 52$ carrier ($N = 64$, $E = 16$) system operating at 5GHz over a 20MHz bandwidth using various constellations ranging from BPSK to 64 QAM. Demodulation is coherent, and the channel can be estimated at the beginning of a burst ($l = 0$), thanks to 2 OFDM training symbols. Moreover, known pilot symbols are transmitted on subcarriers $m = -26; -7; 7; 26$ all along the burst. Therefore, the joint ML estimation can be directly applied on the pilot carriers. As l increases, the linear approximation quickly becomes incorrect and only two-dimensional optimization can be performed.

An interesting feature of HL2 is that linked sources are mandatory at the transmitter and at the receiver. In other words, carrier and clock frequencies are generated from the same crystal. If the carrier frequency is compensated by controlling this reference oscillator, then the clock is simultaneously adjusted. As mentioned in section II, the carrier frequency is estimated so that $f_e < 2\%$. This represents a relative carrier accuracy of ≈ 1 ppm, and the same relative clock accuracy, since both frequencies are proportional. With such a precision, it can be verified on figure 2 that the CPE still needs compensation, but not the VPE. Therefore, in expression (5), parameter y can be replaced by the VPE estimate $\hat{B}$ ($\hat{B} = 0$ if analog oscillator control is implemented). One can verify on figure 4 that if the estimation relies only on a single OFDM symbol observation, the ML estimator of equation (8) is the best one. In comparison, the LLS estimator of equation (7) performs poorly. Otherwise when $L > 1$, one dimensional optimization must be applied to expression (5) with $y = \hat{B}$. Note that i) the optimization algorithm can be refined in order to get best performance especially at the frame start. Here a very basic one (2 iterations $\forall l$, $\forall SNR$) has been simulated ii) in HL2, contrary to DAB, ML estimation provides a significant gain (up to 1dB) enabling an almost perfect rotations compensation.

Although linked sources have been adopted in the HL2 standard, we show on figure 5 that they are not essential for the system. In this case, VPE cannot be derived from the carrier frequency estimate and must be estimated together with CPE. Both CPE and VPE can be digitally compensated in the frequency domain by a single complex multiplication per FFT output. Concerning estimation, joint ML and joint LLS are competing but the latter can be dismissed because of its poor performance. Simulations have been run with the same carrier frequency offset as above, and assuming the worst possible clock FO: 40 ppm. Once again, only two iterations per dimension are considered. Results are presented on figure 5. As expected only the new joint ML algorithm allows an almost perfect FO correction.

V. Conclusion

In this paper, Maximum Likelihood estimation has been applied to solve the problem of carrier and clock tracking in OFDM systems. From a single criterion –the maximization of the joint Maximum likelihood– several new estimators have been derived depending on the considered system features (coherent/differential, etc...). Simulations were performed in the DAB and HIPERLAN/2 contexts. In both systems, especially in the latter, Maximum Likelihood estimation enables an all-digital compensation of frequency offsets and improves the BER performance, with a potential 1 dB gain and affordable complexity.

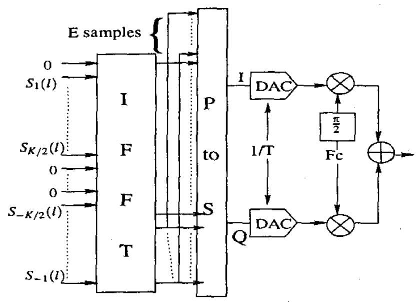

Figure 1: OFDM transmitter

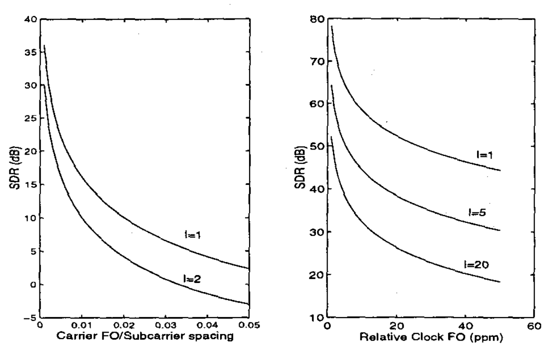

Figure 2: Degradation of the SDR due to phase drifts after l symbols

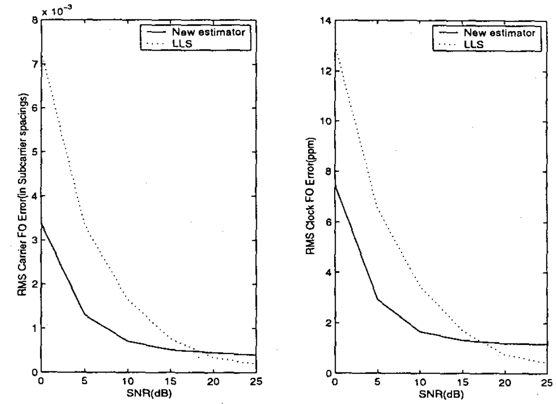

Figure 3: RMS carrier (left) and clock (right) estimation error of the new estimator vs joint LLS estimator (DAB mode 1, typical urban channel, averaged on 4 symbols

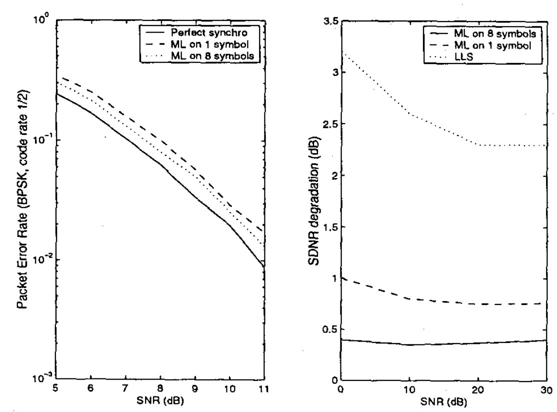

Figure 4: Performance of the carrier only ML estimator in HL2

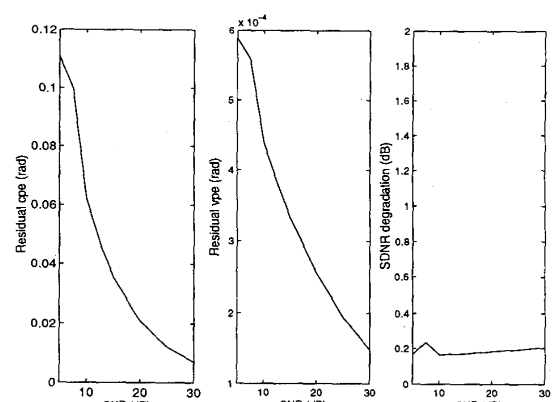

Figure 5: Performance of the joint ML estimator in HL2 ($L = 16$)

VI. References

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