A MMSE Successive Interference Cancellation Scheme for a New Adjustable Hybrid Spread OFDM system

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Abstract - The effects of uniform spreading in OFDM-CDMA systems is the harmonization at the reception of the Signal to Noise Ratio between the sub-bands which prevents the good performance of Successive Decoding algorithms. This paper proposes a new Hybrid Spread OFDM (SOFDM) transmission scheme in which the spreading of the information is adjustable and not uniform along the carrier (frequency selective). Moreover a MMSE version of the V-BLAST Successive Interference Cancellation algorithm suited for this hybrid modulator is derived. The performance of the combination of SHOFDM and MMSE V-BLAST is shown to both outperform COFDM and conventional and classical iterative detection algorithms for SOFDM in the realistic scenario of the 5GHz HiperLAN/2 system.

I. Introduction

A multi-carrier OFDM system [1] using a Cyclic Prefix (CP) for preventing inter-block interference is known to be equivalent to multiple flat fading parallel transmission channels in the Frequency Domain (FD). In such a system, the information sent on some carriers might be subject to strong attenuations and could be unrecoverable at the receiver. This has motivated the proposal of more robust transmission schemes combining the advantages of CDMA with the strength of OFDM known as OFDM-CDMA [3]., in which the information is spread across all the carriers by a pre-coding unitary matrix (e.g. the Walsh-Hadamard: WH transform).

This combination increases the overall frequency diversity of the modulator, so that unreliable carriers can still be recovered by taking advantage of the subbands enjoying a high Signal to Noise Ratio (SNR). Although originally proposed for a multiuser access scheme, this concept can be used in a broader sense in all single user OFDM systems and is referred in the sequel as Spread OFDM (SOFDM).

Due to the inter-carrier interference generated by the spreading, the frequency domain channel transfer function of a single antenna SOFDM system can be modeled using a full MIMO flat fading (scalar) matrix. Actually this is an assumption often made in multiple emitting and receiving antenna communications and already exploited in V-BLAST. Here, the advantage of OFDM systems with CP is that it validates the above assumption *even for channels with memory*.

Thus this paper presents both an extension of the Successive Interference Cancellation (SIC) algorithm V-BLAST [8] in a spirit similar to that for CDMA multiuser detection [7] and a new spreading method that combined with this new algorithm, provides an additional performance gain.

SIC algorithms relies on a sequential detection of the received block. At each step, one symbol is detected before being subtracted from the received block. This introduces successively freedom degrees which enable to reduce the noise/interference influence for the next users to be detected and therefore increases the reliability of the decision process.

But for performing a good interference cancellation, due to the underlying feedback mechanism involved in the successive detection, such methods should decode first the reliable carriers enjoying a greater SNR and then the most corrupted ones. Unfortunately with a WH spreading, all the carriers share the same SNR resulting in practice to marginal performance gain when applying SIC approaches.

In order to overcome this problem, and achieve higher gains, we propose in this paper a new *adjustable* hybrid modulator scheme adopting a non uniform spreading along the carriers (frequency selective) instead of the classical uniform one, achieving a tradeoff somewhere between flat WH-OFDM and plain OFDM.

The purpose of the paper is thus twofold:

1. to propose a new adjustable flexible hybrid spreading modulator - referred in the following as SHOFDM - suited for combination with SIC techniques (section II);
2. to derive and apply to SHOFDM a new MMSE version of the original ZF V-BLAST algorithm (section III).

Section IV finally illustrates how the Hybrid SOFDM transmission scheme can benefit from the improved new

Successive Detection (SD) algorithm. This is achieved by assessing how it performs in comparison to classically equalized SOFDM or plain Coded OFDM (COFDM) in the ETSI BRAN HiperLAN/2 (HL2) 5GHz local area broadband wireless system context (similar to IEEE802.11a) for both a convolutionally coded and uncoded scenario. It is shown that SHOFDM combined to MMSE V-BLAST decoding outperforms current state of the art iterative detection methods such as the one proposed in [6] with one iteration.

II. New Adjustable Hybrid SOFDM transceiver model

II.1. Notations and general SOFDM digital model

In the following, upper (lower boldface) symbols will be used for matrices (column vectors) whereas lower symbols will represent scalar values, $(.)^T$ will denote transpose operator, $(.)^\star$ conjugation and $(.)^H = \left((.)^T\right)^\star$ hermitian transpose. $I_{P\times P}$ will represent the $P \times P$ identity matrix.

Overall system model: Since a N carrier OFDM system [1] using a CP is equivalent in the FD to N flat fading parallel transmission channels, the baseband discrete-time block equivalent model of a SOFDM system can be depicted in figure 1. Actually the $N \times 1$ received block vector $\mathbf{r} = (r_1, \cdots, r_N)^t$ can be expressed in the FD as a function of the emitted symbol $\mathbf{s} = (s_1, \cdots, s_N)^t$ and additive noise $\mathbf{b} = (b_1, \cdots, b_N)$ vectors using a MIMO flat fading channel matrix M:

$$\mathbf{r} = M\mathbf{s} + \mathbf{b} \tag{1}$$

where M consists in the product of the spreading matrix T (usually a WH transform), which can be interpreted as a source of inter-carrier interference, by the diagonal matrix $D = \mathrm{diag}(c_1, \cdots, c_N)$ of the FD channel attenuations:

$$M = DT = (\mathbf{m}_1, \cdots, \mathbf{m}_N)$$

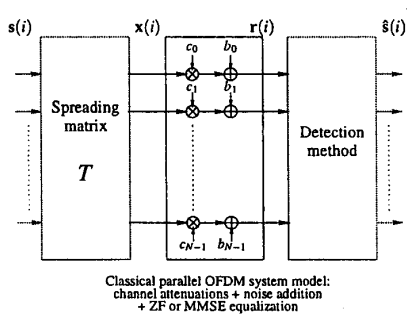

Figure 1: Classical frequency domain SOFDM transmitter

Forward Error Correcting code issues in SOFDM: In classical standardized OFDM systems such as HL2, the incoming input bitstream is first convolutionally encoded with a code rate of R, interleaved and punctured. The resulting bits are then mapped onto a 4 point complex QPSK constellation for forming symbols that are distributed over all the carriers.

Unfortunately, in SOFDM the convolutional encoder cannot be applied prior to the carrier symbol allocation without resulting in an extremely complex Viterbi decoding. Otherwise the metrics calculation could not be processed on a per carrier basis due to the inter-carrier noise correlations introduced by the de-spreading of the received samples and this would exponentially increase the number of states of the Viterbi Algorithm (VA) trellis. This is why for SOFDM, the same coding is applied on each of the carriers independently.

II.2. The hybrid SOFDM scheme

Many methods for retrieving the emitted symbols have already been investigated such as conventional Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) equalization, Maximum Likelihood and iterative decoding (cf.[3] and references therein).

As already mentioned in the introduction, SIC schemes do not couple very well with SOFDM because they require to be able to sort the carriers for performing decisions on the most reliable ones first. By construction, the role of a uniform spreading for SOFDM is to equalize the SNRs between the sub-bands so that no carrier can be candidate for being decoded first when using a successive decoding method. This results in practice in a poor interference cancellation coming from too important error propagation in the feedback (even when using a soft decision mechanism). This phenomenon simply annihilates the benefits of such technique.

On the other hand, in plain OFDM, it is extremely easy to find the reliable carriers due to the difference of SNR affecting the various subbands. However, in this specific case the successive decoding of the components of the received vector is of no interest since the carriers are always assumed to be independent.

These considerations inspired us to propose a new Hybrid scheme combining the strength of SOFDM and OFDM that is suited for successive detection schemes and expected to enhance the performance of decoding methods such as V-BLAST presented section III. The basic idea is to change the nature of the spreading and adopt an adjustable non uniform one along the carriers (frequency selective). That way, a tradeoff between flat WH-OFDM and plain OFDM is performed. The new Hybrid modulator is therefore defined by:

$$T(\theta) = \cos(\theta)I + j\sin(\theta)W$$

where W denotes the Walsh-Hadamard matrix and θ is a parameter used for tuning the modulator. This tunable new modulator deserves a few comments:

- since both I and W are unitary real matrices and $W^H W = W^2 = I$, one can verify that for all θ, $T(\theta)$ is also unitary;
- when $\theta = 0$ the overall transmitter corresponds to a classical OFDM system and when $\theta = \pi/2$ the usual Walsh-Hadamard spread OFDM system is obtained;
- any θ ($\theta \neq 0, \pi/2$) creates a new kind of diversity and improves the successive non linear detection process;
- the above principle can be extended to any real unitary matrix verifying $W^2 = I$.

Thus, we have now at our disposal an adjustable modulator model encompassing both OFDM and WH-SOFDM.

The choice of the relevant θ for a given propagation environment is a research subject in itself and will be detailed in a separate contribution. However, as underlined in section IV, simulations have confirmed that in an uncoded context, $\theta = \pi/4$ is the optimum choice for achieving the best performance on average when combined with successive decoding schemes. In the following a SOFDM scheme where the spreading is performed by $T(\pi/4)$ will be referred as Hybrid SOFDM: SHOFDM.

Note that, since the proposed scheme achieves a "frequency selective" spreading it is important to use both a frequency and time bit interleaving in the coded case.

III. MMSE Successive Interference Cancellation scheme

Taking a closer look to equation 1, one can notice that the overall SHOFDM transceiver transfer function is the matrix M. Therefore all classical detection schemes based on a MIMO flat fading channel model can be applied. Actually this is an assumption made for multiple antenna algorithms and is exploited in the V-BLAST approach [8]. The goal of this section is thus both to derive a MMSE version of this ZF-based successive decoding scheme and to apply this algorithm for the decoding of SHOFDM.

III.1. Successive detection algorithm

The algorithm relies on a sequential detection of the received block. At the first step of the method, a wiener equalization of matrix M is performed by matrix $G_1 = (M^H M + \sigma^2 I)^{-1} M^H$. Then the carrier k_1 enjoying the highest Signal to Interference plus Noise (SINR) is decoded. Assuming a perfect decision, the resulting estimated symbol $\tilde{s}_{k_1}$ is subtracted from the vector of received samples in the following manner: $\mathbf{r}_2 = \mathbf{r}_1 - \tilde{s}_{k_1}\mathbf{m}_{k_1}$. This introduces one degree of freedom for the next canceling vector choice which enables to reduce the noise plus interference influence for increasing the decision process reliability. The complete detection algorithm can thus be summarized as follows:

$$\begin{aligned}
&\text{initialization:} && (2)\\
&i \leftarrow 1 && (3)\\
&\mathbf{r}_1 = \mathbf{r} && (4)\\
&G_1 = (M^H M + \sigma^2 I)^{-1} M^H && (5)\\
&k_1 = \text{argmax}_j(\text{SINR}_j^{(1)}) && (6)\\
&\text{recursion:} && (7)\\
&\mathbf{w}_{k_i} = \mathbf{g}_{k_i}^{(i)} && (8)\\
&y_{k_i} = \mathbf{w}_{k_i}\mathbf{r}_i && (9)\\
&\tilde{s}_{k_i} = Q(y_{k_i}) && (10)\\
&\mathbf{r}_{i+1} = \mathbf{r}_i - \tilde{s}_{k_i}\mathbf{m}_{k_i} && (11)\\
&G_{i+1} = (M_i^H M_i + \sigma^2 I)^{-1} M_i{}^H && (12)\\
&k_{i+1} = \text{argmax}_{j \neq k_1, \cdots, k_i}(\text{SINR}_j^{(i+1)}) && (13)\\
&i \leftarrow i+1 && (14)
\end{aligned}$$

Let us define and justify now the remaining unexplained variables involved in the algorithm.

Be $L = (k_1, \cdots, k_N)$ the permutation of $(1, \cdots, N)$ specifying the order in which the components of the transmitted symbol vector $\mathbf{s}$ are extracted.

The Wiener filtering matrix G_i at iteration i is defined as:

$$G_i = \text{argmin}_W \|W^H \mathbf{r}_i - \mathbf{s}_i\|^2 = (M_{i-1}^H M_{i-1} + \sigma^2 I)^{-1} M_{i-1}^H$$

where matrix M_i denotes the matrix obtained by zeroing columns $(k_1, \cdots, k_{i-1})$ of M. Recall that $\mathbf{m}_{k_i}$ represents the k_ith column of M while $\mathbf{g}_{k_i}^{(i)}$ is the k_ith row of G_i at step i.Q represents the decision process.

The optimal selection of $\mathbf{g}_{k_i}^{(i)}$ as well as the choice of Q deserves some explanations.
It is shown in [5] that the distribution of the residual interference-plus noise (SINR) at the output of a linear MMSE filter can be considered as Gaussian. Therefore, we will assume that the output $y_j^{(i)}$ of the MMSE filter can be modeled at each step i of the algorithm by:

$$y_j^{(i)} = a_j^{(i)} s_j + n_j^{(i)}$$

where $a_j^{(i)}$ is the amplitude of the jth symbol at iteration i and $n_j^{(i)}$ is a gaussian noise of law $N(0, \sigma_j^{(i)^2})$. The parame-

ters $a_j^{(i)}$ and $\sigma_j^{(i)}$ are given by:

$$\begin{aligned} a_j^{(i)} &= \mathbf{g}_j^{(i)}\mathbf{m}_j \\ \sigma_j^{(i)^2} &= \mathbf{g}_j^{(i)}\mathbf{g}_j^{(i)^H}\sigma^2 + \sum_{\forall p \neq k_1,\cdots,k_i}^{n} \mathbf{g}_j^{(i)}\mathbf{m}_p E(|s_p|^2) \end{aligned}$$

The SINR per symbol is thus expressed as:

$$\mathrm{SINR}_j^{(i)} = \frac{a_j^{(i)^2} E(|s_j|^2)}{\sigma_j^{(i)^2}}$$

Note that the chosen vector $\mathbf{g}_j^{(i)}$(equation 8) corresponds to the one leading to the highest SINR value.

Once the symbol is retrieved (equation 9), a decision is made modeled by operator Q (equation 10). Instead of performing a hard sign decision, it is often better to use for Q a "soft" one using the hyperbolic tangent non linear detector whose argument is weighted by an estimation of the SINR. Such a detector is optimum in the MSE sense and closely approximates the sign function when the SINR is high enough. Such a modification of the cancelling mechanism is appealing since it tends to attenuate the effect of unreliable decisions when a low SINR occurs [4]. Thus the expression of $\tilde{s}_{k_i}$ in the QPSK constellation case is:

$$\tilde{s}_{k_i} = \frac{1}{\sqrt{2}}\left(\tanh\left(\frac{\mathrm{real}(y_{k_i})}{\sigma_{k_i}^{(i)^2}}\right) + j\tanh\left(\frac{\mathrm{imag}(y_{k_i})}{\sigma_{k_i}^{(i)^2}}\right)\right)$$

Finally, in the uncoded case, once all symbols have been iteratively retrieved, a hard decision is performed on the resulting vector $\mathbf{y} = (y_1, \cdots, y_N)^t$.

III.2. Convolutional Coding

Applying this MMSE successive decoding algorithm on a convolutional coded SHOFDM system requires some customization of the usual calculation of Viterbi metrics which are detailed below. Recall that the soft value of the retrieved QPSK symbol has the following form: $y_{k_i} = a_{k_i}^{(i)} s_{k_i} + n_{k_i}^{(i)}$ which is processed by the Viterbi decoder after deinterleaving. Since each carrier is independently convolutionally coded, the resulting metric can be straightforwardly derived from the conditional probability density function that y_{k_i} is received knowing that s_{k_i} has been transmitted:

$$p(y_{k_i} \mid s_{k_i}) = \frac{1}{\sqrt{2\pi\sigma_{k_i}^{(i)^2}}} exp\left(\frac{|\, y_{k_i} - a_{k_i}^{(i)} s_{k_i} \,|^2}{\sigma_{k_i}^{(i)^2}}\right)$$

Hence the metric to be applied in the VA for each symbol is:

$$\frac{|\, y_{k_i} - a_{k_i}^{(i)} s_{k_i} \,|^2}{\sigma_{k_i}^{(i)^2}}$$

In SOFDM, all the metrics are equally weighted due to the SNR averaging process performed by W. This important remark enables to intuitively understand why the proposed hybrid scheme is expected to outperform coded SOFDM. Actually, due to the interleaving process, SHOFDM will give birth to different metrics from one symbol to the other. Therefore, the VA will benefit from this diversity and increase the reliability of its decision process by correcting corrupted symbols exploiting the good ones through the memory introduced by the coder.

IV. Simulation results

This section compares the performance of the new MMSE V-BLAST Successive Detection (SD) algorithm applied both on SHOFDM and SOFDM systems to classical OFDM and SOFDM schemes in a coded and uncoded scenario.

Simulations have been performed in the HL2 system context using QPSK constellations. HL2 is a $N = 64$ carrier broadband wireless system operating in the 5GHz band over 20MHz (equivalent to IEEE802.11) using a 16 samples Cyclic Prefix. The BER versus Eb/No of the various detection algorithms are illustrated fig. 3 and fig. 2, based on Monte Carlo simulations, with each trial corresponding to a different realization of the typical 5GHz wireless Channel Model A specified by ETSI [2] and assuming perfect synchronization and channel knowledge.

Uncoded Case: Fig. 2 underlines the gain achieved with the new system over competing techniques in the uncoded case. When using a MMSE filter, the SHOFDM curve is between the plain OFDM (note that ZF and MMSE equalized OFDM give the same results for QPSK constellations) and Spread-OFDM. This is quite justified since a linear detector is used. On the other hand, for a BER of 10^{-4}, a gain of 4.5 dB is achieved over MMSE SOFDM when MMSE Successive Detection is used on the hybrid modulator. For the same detection technique, SHOFDM outperforms classical WH-SOFDM by a gain of 3 dB at BER 10^{-4}.

The lower bound theoretical curve corresponds to a WH-spreading with no interference: i.e. a diagonal channel matrix where each coefficient is the summation of the squared frequency channel coefficients.

A derived version of the iterative soft block decision feedback equalizer proposed in [6] has also been compared to our new detection scheme. For these methods, it is known that the highest gain is achieved only after the first iteration. Therefore only one iteration has been plotted (curve Iterative Decoding: ID SOFDM of fig. 2). Still, a gain of 3dB is achieved with our new system. Anyway, for higher BER, greater gains are always achieved.

Coded Case: It is shown in [4] that a trade-off between Channel Coding and Spreading in Multi-Carrier CDMA

Systems is to be considered. Actually spread OFDM outperforms COFDM for code rates higher than $1/2$: $R > 1/2$ while there is no significant difference for code rates $R < 1/2$. An explanation of this phenomenon is that, associated with interleaving, the high redundancy of low code rates already performs a kind of spreading by linking the various carriers through the memory introduced by the CC which exploits all the diversity of the channel.

Therefore in simulations, a rate R=3/4 with constraint length K=7 Convolutional Coder (CC) is applied before the QPSK mapping which by the way corresponds to a real HIPERLAN/2 transmission mode. Note that each carrier is decoded with the Viterbi Algorithm. As shown in fig. 3, the parameter θ of the Hybrid modulator must be carefully chosen according to the coding rate. For $R = 3/4$, it seems that the trade off between OFDM and SOFDM leads to $\theta \approx \pi/6$ but this subject still deserves more research for drawing accurate conclusions. For this modulator a further gain of 3dB is achieved over MMSE SOFDM at BER of 10^{-4} while 2dB are gained over MMSE SD SOFDM.

V. Conclusion

In this contribution, we have proposed and applied a new MMSE successive interference cancellation based on the V-BLAST algorithm to a new Hybrid adjustable OFDM modulator. The new diversity provided by the modification of the spreading nature provides means for great improvements over classical OFDM and SOFDM schemes at a cost of an increased arithmetical complexity for the decoding.

A gain of more than 4.5 dB is achieved over MMSE V-BLAST WH-SOFDM for a BER of 10^{-4} in the uncoded case. These results, simulated in a realistic environment (HiperLAN/2), confirm that Hybrid OFDM is a promising transmission technique over frequency wireless selective fading channels.

Still further studies should be conducted on the optimal choice of the hybrid spreading parameter θ for a given channel and code rate. A simplification, in terms of complexity, of the MMSE successive interference cancellation scheme is also under investigation in the SHOFDM case.

VI. References

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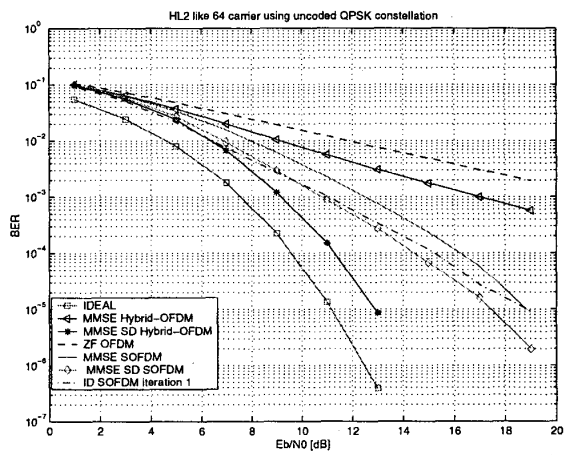

Figure 2: Uncoded BER

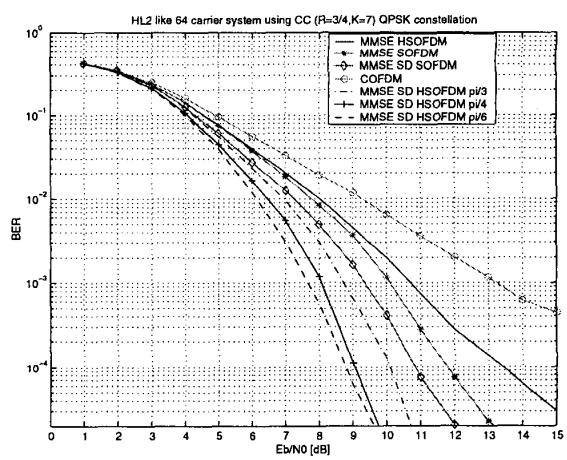

Figure 3: Convolutionally Coded BER